

Edge-Unfolding Orthogonal Polyhedra is Strongly NP-Complete

Zachary Abel¹ Erik D. Demaine²

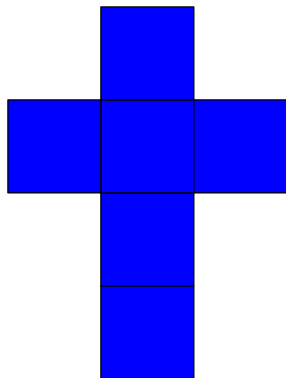
¹MIT Department of Mathematics

²MIT CSAIL

CCCG 2011

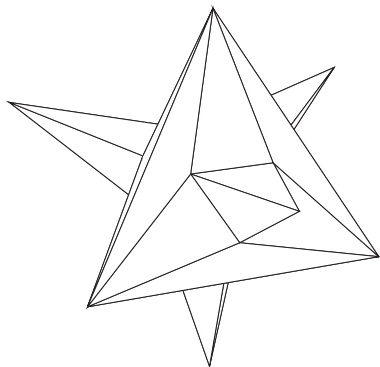
Background on Edge-Unfolding

- **(General) Unfolding:**
cut and fold flat without overlap
- **Edge-Unfolding:**
cut only on edges



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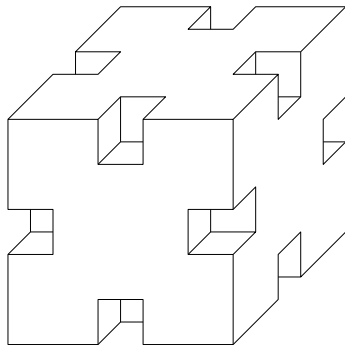
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cut only on edges
- Edge-unfolding does not always exist



[BDEKMS, CCCG '99]

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[BDDLOORW, CCCG '98]

Complexity of Edge-Unfolding

- Decision question
- In NP? Not known!
 - ▶ Needs high numerical precision (radicals)
- *Orthogonal* edge-unfolding is in NP

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Our result:

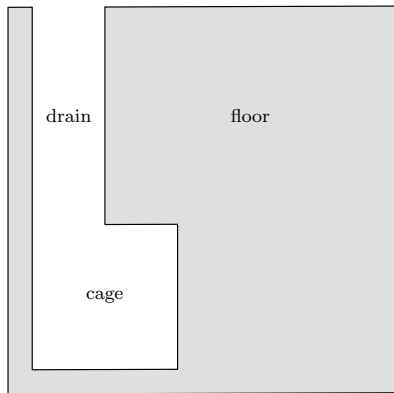
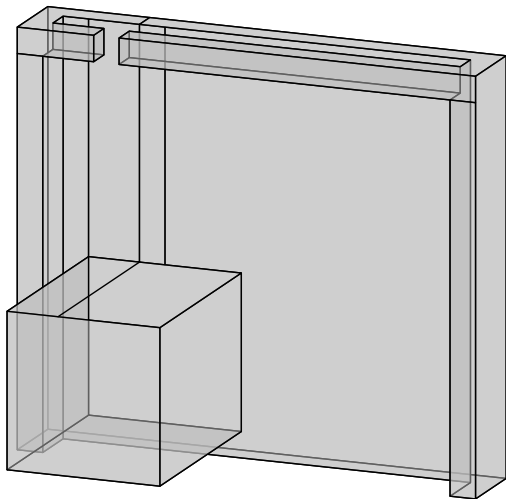
- Precision is not the only barrier

Theorem

Edge-unfolding is strongly NP hard even if the polyhedron must be orthogonal, topologically convex, and of genus zero.

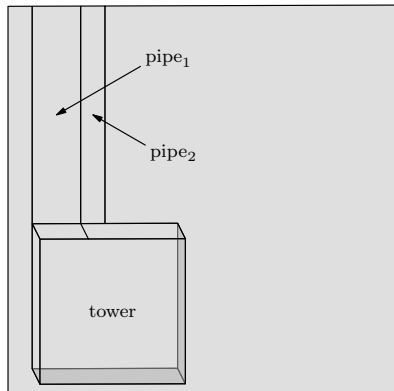
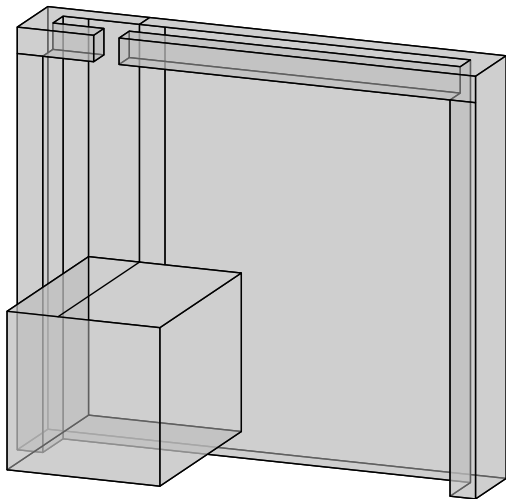
A New Ununfoldable Polyhedron

Orthogonal and Topologically Convex



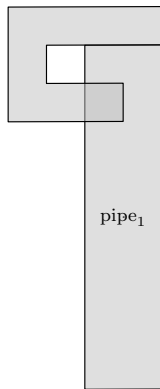
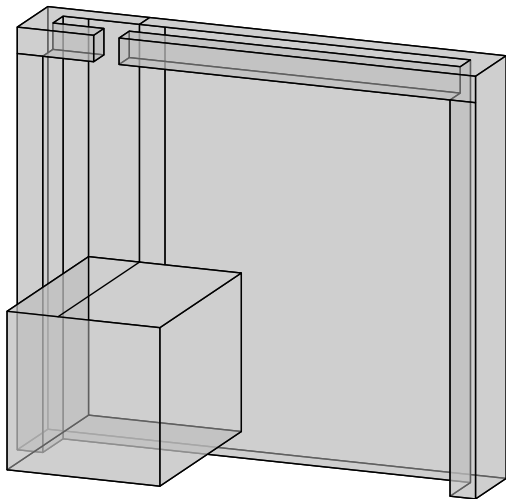
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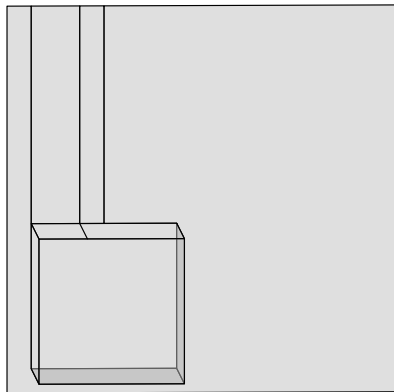
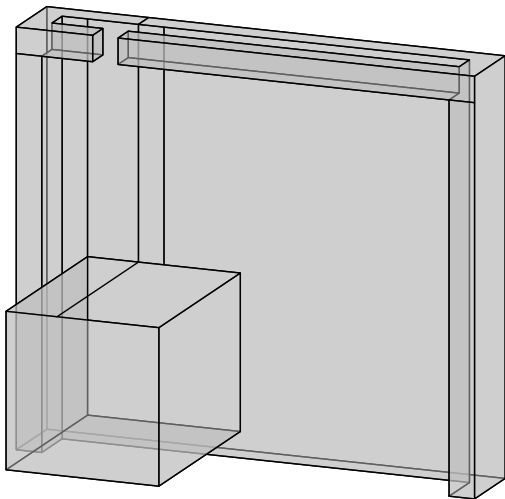
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U-shaped polygons must be removed from floor and pipes.

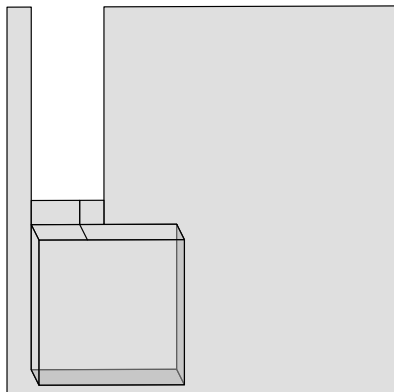
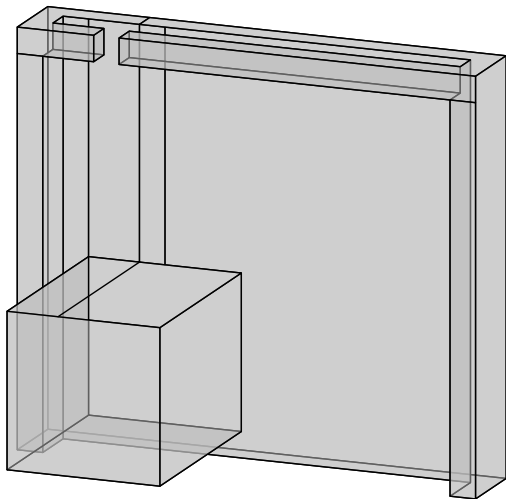
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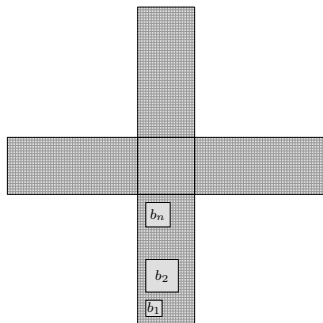
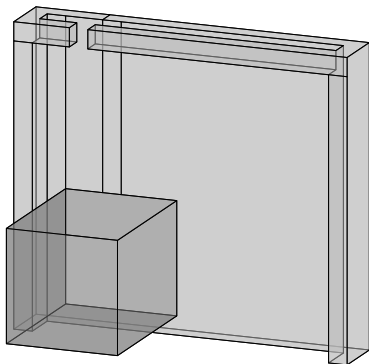
Too many big faces on tower to fit in cage

Reduce from Square Packing

Square Packing Problem

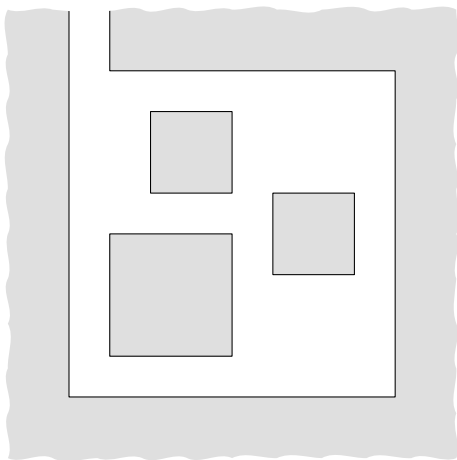
Can n squares of (integer) side-lengths a_1, \dots, a_n be orthogonally packed into a square of side-length d ?

Cover the tower with large *bricks* b_1, \dots, b_n and tiny *filler material* so the bricks must unfold inside the cage.



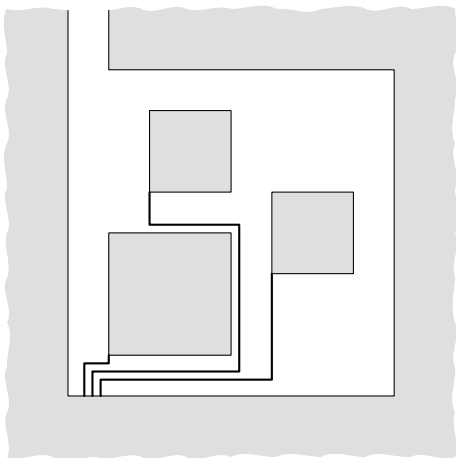
Making a Square Packing Sufficient

- Need connectivity
- Need room for unused filler



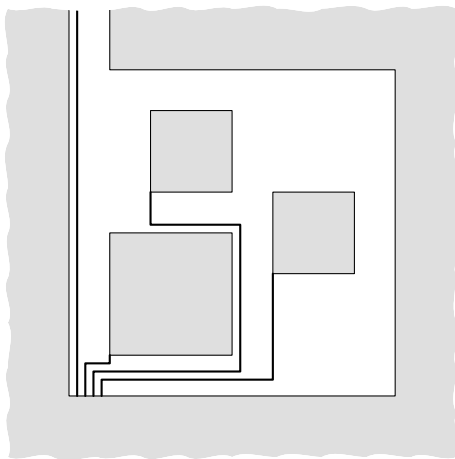
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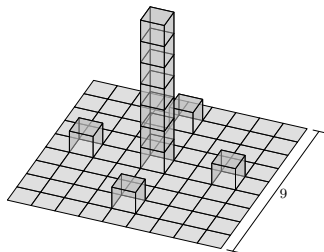
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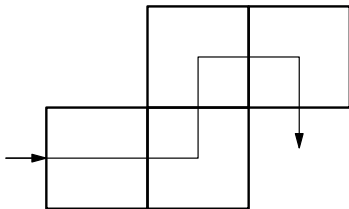
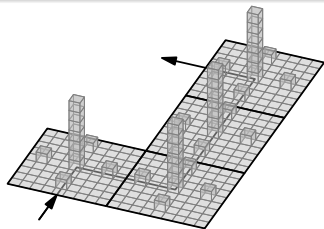
Universal Filler Material

- **Atom**: this polyhedral surface
- **Flatom**: a 27×27 square



Theorem

Any path of n edge-connected atoms can be edge-unfolded inside any path of n edge-connected flatoms, connecting to the initial and final edges at the midpoint (or one unit away depending on parity).



Universal Filler Material (Ctd.)

